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Linear Dilatons, NS5-branes and Holography

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Abstract: We argue that vacua of string theory which asymptote at weak coupling to linear dilaton backgrounds are holographic. The full string theory in such vacua is "dual" to a theory without gravity in fewer dimensions. The dual theory is generically not a local quantum field theory. Excitations of the string vacuum, which can be studied in the weak coupling region using worldsheet methods, give rise to observables in the dual theory. An interesting example is string theory in the near-horizon background of parallel NS5-branes, the CHS model, which is dual to the decoupled NS5-brane theory ("little string theory"). This duality can be used to study some of the observables in this theory and some of their correlation functions. Another interesting example is the "old" matrix model, which gives a holographic description of two dimensional string theory.

Keywords: p-branes, D-branes, String Duality, Superstring Vacua.

Contents

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1. Introduction and discussion

It has been suggested by 't Hooft [1] and Susskind [2] (see also [3]) that any consistent quantum theory of gravity must be holographic, *i.e.* the number of degrees of freedom in any spatial domain is finite, proportional to the area of the boundary of the domain in Planck units. This is unlike standard local quantum field theories in two respects. First, the number of degrees of freedom of quantum field theory is proportional to the volume of the system rather than the area of its boundary. Second, continuum quantum field theories have an infinite number of degrees of freedom per unit volume. The latter difference is presumably responsible for the finiteness of string theory, and appears like a built-in cutoff. The first is more surprising. It basically states that the degrees of freedom in a certain region "live" on the boundary of the region rather than in the interior. Equivalently, in any generally covariant theory it is difficult to define local observables, and therefore it is natural to assume that there are no such observables. The only observables should exist on the boundary.

In some vague sense this reduction by one dimension is quite familiar in string theory. It is widely believed that string theory, as a theory which describes all particles and interactions, has only on-shell information. The theory cannot be probed by sources which are not within the theory itself, and hence we cannot probe it off-shell. Therefore, in string theory we usually compute S-matrix elements rather than Green functions. In $d+1$ space-time dimensions the on-shell momenta have only d independent components while the off-shell momenta have $d+1$ components. This suggests that an on-shell theory like string theory in $d+1$ dimensions can be equivalent to an off-shell theory in d dimensions. In particular, Polyakov suggested that a field theory like QCD in d dimensions can be equivalent to a $d+1$ dimensional string theory [4].

Recently these ideas have been made much more concrete. String theory and M-theory in anti-de-Sitter space [5] beautifully demonstrate holography [6]. The theory on the boundary is in this case a local quantum field theory, whose observables are correlation functions of local operators. In the bulk theory they describe the response of the theory to disturbances at infinity [7, 8].

The purpose of this note is to study a class of string backgrounds which exhibit holography but whose boundary dynamics is in general not local. Specifically, we will discuss linear dilaton backgrounds which asymptote, as a space-like coordinate $\phi \to \infty$, to spacetimes of the form $\mathcal{M} \times \mathbb{R}^{d,1}$ where $\mathcal{M}$ is a compact space and ϕ is one of the coordinates in $\mathbb{R}^{d,1}$. The string frame metric and string coupling asymptote to

$$\begin{aligned} ds^2_{string} &= dx^2 + d\phi^2 + ds^2(\mathcal{M}) \\ g_s^2 &= e^{-Q\phi}\,, \end{aligned} \tag{1.1}$$

where x is a coordinate on $\mathbb{R}^{d-1,1}$ and the string metric on $\mathcal{M}$ is independent of x and ϕ. We will also comment on the case where some of the coordinates of $\mathbb{R}^{d-1,1}$ are compactified on a torus.

We propose that any string background that behaves asymptotically as (1.1) is equivalent to a lower dimensional off-shell theory without gravity whose observables live at the boundary $\phi \to \infty$. Off-shell physical observables in the "boundary" theory are identified with on-shell physical excitations in the string background (1.1). Note that this proposal is in agreement with a known property of string vacua which asymptote to (1.1) [9]: the profiles of physical string excitations, *e.g.* those described on the worldsheet by BRST invariant vertex operators, are non-normalizable and supported (typically exponentially in ϕ) at $\phi \to \infty$.

Green functions in the off-shell boundary theory are identified, as in the AdS/CFT correspondence, with on-shell amplitudes in string theory. Perturbatively they are given by worldsheet correlation functions of the corresponding physical vertex operators. It is well known that generic worldsheet correlation functions are sensitive to the spacetime background at finite ϕ and not just to the asymptotic form (1.1). On the worldsheet this is the statement that higher genus contributions to correlation functions become more and more important as $\phi \to -\infty$. In spacetime this can be seen by analyzing the metric (1.1) in the Einstein frame,

$$ds^2_{Einstein} = e^{\beta\phi}(dx^2 + d\phi^2 + ds^2(\mathcal{M}))\,, \tag{1.2}$$

where the positive number β depends on Q, d and the dimension of $\mathcal{M}$. As $\phi \to \infty$ the distances between points at fixed x in $\mathbb{R}^{d-1,1}$ diverge, as in the AdS examples. Therefore, disturbances on the boundary have to propagate to the bulk before they can interact. This is a necessary condition for holography [10]. Equivalently, in the string frame the distances remain finite but the string interactions vanish as $\phi \to \infty$. Again, signals have to propagate to the bulk in order to interact.

To summarize, the description of the bulk theory (1.1) is useful near $\phi \approx \infty$, where the string theory is weakly coupled and excitations, such as BRST invariant vertex operators and D-branes, can be studied using worldsheet methods. Holography relates these excitations to observables in the boundary theory. To compute correlation functions some information about the strong coupling region $\phi \to -\infty$ is needed.

There are several known classes of string vacua which asymptote to (1.1). Different vacua utilize different mechanisms for regulating the divergences in the strong coupling region. In Liouville theory (for reviews see [9, 11, 12]) a tachyon condensate, a potential on the worldsheet, makes it harder for the strings to propagate to the strong coupling region. In the two dimensional black hole [13, 14], which is the $SL(2, \mathbb{R})/U(1)$ coset theory, the spacetime topology at finite ϕ is modified and the string coupling is bounded. Finally, as we will see below, in the theory of NS5-branes the resolution of the strong coupling singularity cannot be understood using worldsheet methods. String duality can be used to show that in some cases the low energy theory becomes weakly coupled in other variables.

When some of the coordinates x in (1.1) are compactified on a torus, the underlying string theory enjoys a T-duality symmetry, implying a symmetry between momentum and winding modes. Therefore, the notion of locality in the boundary theory becomes confusing. Furthermore, because of this T-duality the boundary theory cannot have a unique energy momentum tensor. Such arguments were used in [15] to argue that the theory of NS5-branes ("little string theory") is not a local quantum field theory. Here we see that this is a common feature of the boundary theories of all backgrounds of the form (1.1) (with sufficiently large d).

As mentioned above, our discussion applies to some string backgrounds that were studied in the past. One example is the "old matrix model" (for a review see [12]), which has several interpretations. It describes the quantum mechanics of $N \times N$ matrices in the limit $N \to \infty$. It can also be thought of as a theory of two dimensional gravity. A third interpretation arises from interpreting the two dimensional gravity theory as the worldsheet dynamics of a string. This leads to string theory in a $1+1$ dimensional spacetime with the Liouville field ϕ playing the role of a space coordinate. The dilaton of this spacetime theory is linear in ϕ.

In modern language we can say that the equivalence of the large N matrix model to two dimensional gravity coupled to $c = 1$ matter (or equivalently $1+1$ dimensional string theory) is an example of the holography proposed above. The matrix model gives a holographic description of two dimensional string theory.[1] The observables of the theory can be described in terms of the matrices and we can compute their Green functions. These are related to the S-matrix elements of the bulk spacetime theory which can be computed using standard worldsheet methods (vertex operators).

[1]This was suggested by T. Banks several years ago.

The equivalence of the matrix model and $1+1$ dimensional string theory provides a rather simple example of holography. The arguments above regarding non-locality of the "boundary" theory are inapplicable here because of the low spacetime dimension; indeed the boundary theory is standard matrix quantum mechanics. Similarly, this example is not well suited for studying the relation between a bulk theory of gravity and a boundary theory without gravity since the gravitational sector of the bulk theory (which consists of certain "discrete states") essentially decouples from the dynamics.

A richer set of holographic theories of the sort discussed here was constructed in [16]. In the notations of (1.1), the compact manifold $\mathcal{M}$ in the specific case discussed in [16] is a circle[2] and the dimension of the boundary d can take the values $d = 2, 4, 6$. The strong coupling singularity at $\phi \to -\infty$ is removed as in Liouville theory by turning on a worldsheet superpotential. Consider for example the theory with $d = 4$. It is invariant under eight supercharges which anticommute to translations in x, but not in ϕ (of course, translations in ϕ are not a symmetry) or along the circle. It is natural to conjecture that the full string theory which is naively six dimensional in this case is equivalent to a four dimensional off-shell theory without gravity with $\mathcal{N} = 2$ SUSY.

A possible candidate for the "boundary" theory is the decoupled theory of an NS5-brane with worldvolume $\mathbb{R}^{3,1} \times \Sigma$ with Σ a Riemann surface that can be obtained as follows. Start with a configuration of N D4-branes suspended between two parallel NS5-branes in type IIA string theory (see [17] for a review of the physics of such configurations). We can for example take all the branes to be infinite in (x^0, x^1, x^2, x^3); the fivebranes are further extended in (x^4, x^5) and the fourbranes are suspended between them along the x^6 axis. This configuration preserves eight supercharges and describes at low energies a four dimensional $\mathcal{N} = 2$ SYM theory with gauge group $SU(N)$. Taking the IIA string coupling to infinity, the above brane configuration turns into an M5-brane in eleven dimensions with worldvolume $\mathbb{R}^{3,1} \times \Sigma$ where Σ has been determined in [18]. If we now compactify one of the coordinates x^7, x^8, x^9 on a small circle we get a similar configuration in IIA string theory. The resulting theory on the NS5-brane with worldvolume $\mathbb{R}^{3,1} \times \Sigma$ is not a local QFT,[3] however it reduces in the infrared to $\mathcal{N} = 2$ SYM with gauge group $SU(N)$.

This theory has a few features in common with the $d = 4$ theory constructed in [16]. The global symmetries of both are naively $U(1) \times U(1)$. In the theory of the NS5-brane one $U(1)$ corresponds to rotations in the (x^4, x^5) plane while the other is the $SO(2)$ subgroup of rotations of (x^7, x^8, x^9) unbroken by the construction above. The second $SO(2)$ symmetry is enhanced to $SO(3)$ in the extreme infrared limit of the theory on the fivebrane. In the construction of [16] the $U(1) \times U(1)$ symmetry corresponds to momentum and winding on the circle. One of the $U(1)$ factors is actually broken in both theories. In the theory of the fivebrane it is broken by quantum effects (*i.e.* at finite QCD scale Λ). In the theory of [16] it is broken by the worldsheet superpotential

[2]There are many possible generalizations of the models of [16] for which the manifold $\mathcal{M}$ is different.
[3]A similar construction is described in [19].

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that stops the theory from running to strong coupling. Thus we are led to identify the worldsheet cosmological constant with the QCD scale, Λ. The other $U(1)$ remains unbroken in both theories (and as mentioned above should be enhanced to $SU(2)$ in the extreme IR).

Another possibility for regulating the strong coupling singularity at $\phi \to -\infty$ in the theories of [16] is to replace the cylinder $\mathbb{R} \times S^1$ with a Liouville-like superpotential by the supersymmetric $SL(2)/U(1)$ coset, which changes the topology to that of a cigar and removes the strong coupling region. The symmetry structure of the resulting string theory is the same as that discussed above.

In the remaining sections we will focus on the specific example of string theory in the near-horizon geometry of parallel NS5-branes in type II string theory,[4] that was studied by Callan, Harvey and Strominger (CHS) [20]. We will argue that the theory with vanishing asymptotic string coupling is dual to the non-local six dimensional theory without gravity that governs the dynamics of NS5-branes at vanishing string coupling [15]. Section 2 contains rudiments of the relevant classical supergravity solution. Section 3 contains an analysis of some dynamical issues in this background. In particular we identify the set of short representations of the NS5-brane theory with vertex operators in the weakly coupled string theory regime (the tube of the CHS theory). These are given by primaries of the affine $SU(2)$ on the string worldsheet. It is known that the NS5-brane theories have an $A-D-E$ classification. Our analysis extends the explanation of [21] of the $A-D-E$ classification of affine $SU(2)$ modular invariants.

2. The near-horizon limit of NS5-branes in string theory

The decoupled theories on NS5-branes in type II string theory were first discussed in [15], motivated by the study of compactifications of Matrix theory on high dimensional tori [22]. In [15] it was argued that the theory on N NS5-branes[5] decouples from the bulk in the limit

$$g_s \to 0\,, \qquad M_s = \text{fixed}, \tag{2.1}$$

because the effective coupling on the NS5-brane is M_s, while the coupling to bulk modes behaves as g_s. A DLCQ description of this theory further supported the existence of a consistent theory that is decoupled from the bulk [23, 24].

The existence of a decoupled theory of NS5-branes seemed to be in conflict with previous analyses of this system [20, 25]. In particular, it was pointed out [26] that for an energy density that is finite in string units there is finite Hawking radiation to the CHS tube region of the 5-brane solution, suggesting that the theory does not decouple from the fields in this region.

[4] Our analysis can be easily generalized also to other six dimensional theories, such as the heterotic 5-brane theory.

[5] We take $N \geq 2$ for reasons that will be clarified in the next section.

The proposal in [5] can be used to reconcile the two points of view.[6] According to this conjecture a d dimensional theory without gravity, such as our theory for $d = 6$, may be dual to a higher dimensional theory with gravity. In our case, string or M-theory in the background of the 5-brane solution in the limit (2.1), including the CHS tube, is conjectured to be dual to the decoupled theory on the NS5-branes. In particular, the fields in the tube which arise in the Hawking radiation [26] are interpreted as part of this decoupled theory. Correlation functions of observables in this theory may be defined by setting appropriate boundary conditions at the weak coupling boundary of the CHS background.

In this section we will discuss the classical supergravity solution which arises in the limit (2.1) of the metric of NS5-branes. M theory in this background is conjectured to be dual to the decoupled NS5-brane theory. Parts of this section overlap with [27, 28].

2.1. IIA and M theory 5-branes

We start by discussing NS5-branes of type IIA string theory, which may be viewed as M-theory 5-branes localized on the eleven dimensional circle. Thus, they are described by the metric for N M5-branes at a point on a transverse circle. M-theory has a scale l_p, and the radius of the circle asymptotically far away from the 5-branes will be denoted by R_{11} (we will not be careful about numerical factors). The (asymptotic) string scale is given by $R_{11} l_s^2 = l_p^3$. We are interested in taking the limit (2.1) in which l_p and R_{11} go to zero with l_s kept fixed. l_s will then be the dimensionful parameter of the decoupled theory on the NS5-branes [15]. The near-horizon metric for N overlapping 5-branes in this configuration is given by :

$$ds^2 = H^{-1/3}[dx_6^2 + H(dx_{11}^2 + dr^2 + r^2 d\Omega_3^2)]\,, \tag{2.2}$$

where

$$H = \sum_{n=-\infty}^{\infty} \frac{N l_p^3}{(r^2 + (x_{11} - nR_{11})^2)^{3/2}}\,, \tag{2.3}$$

x_{11} is periodic with period R_{11}, and dx_6^2 is the metric on $\mathbb{R}^{5,1}$. This is the metric for N overlapping 5-branes – the generalization to 5-branes located at different x_{11} positions is straightforward. The supergravity solution involves also a 4-form field strength with N units of flux, which we will not write explicitly.

In the limit (2.1) the natural coordinates to define are such that the tension of a string arising from an M-theory membrane stretched between 5-branes at distances r or x_{11} remains constant. Thus, we choose $U = r/l_p^3$ and $y_{11} = x_{11}/l_p^3$. Both of these coordinates have dimensions of mass squared, and the coordinate y_{11} has a periodicity of $R_{11}/l_p^3 = 1/l_s^2$. In terms of these variables the metric is:

$$ds^2 = l_p^2 \widetilde{H}^{-1/3}[dx_6^2 + \widetilde{H}(dy_{11}^2 + dU^2 + U^2 d\Omega_3^2)]\,, \tag{2.4}$$

[6]Similar ideas have also been suggested by various people, including C. V. Johnson, J. Maldacena and A. Strominger.

with

$$\widetilde{H} = \sum_{n=-\infty}^{\infty} \frac{N}{(U^2 + (y_{11} - n/l_s^2)^2)^{3/2}} \,. \tag{2.5}$$

Except for an overall factor of l_p^2 in front of the metric, it remains finite in this limit. As in the case of a similar factor in the $AdS_5 \times S^5$ metric [5], this l_p will drop out of any physical computations. Below we will study the conjecture that M-theory on the manifold (2.4) is equivalent ("dual") to the six dimensional theory of N NS5-branes in type IIA string theory. Both theories have $(2,0)$ six dimensional SUSY (four supercharges in the **4** of $Spin(5,1)$) and a global $SO(4)$ R-symmetry.

There are two regions where the metric (2.4) simplifies considerably. It is known that distances that are large compared to $\sqrt{N}l_s$ in the six dimensional theory of NS5-branes correspond to the $(2,0)$ SCFT, the extreme IR limit of the NS5-brane theory. Equivalently, we can approach this low-energy limit by taking $l_s \to 0$, which corresponds in (2.4) to small values of U and y_{11} (compared to $1/l_s^2$). In this limit the sum in (2.5) is dominated by the contribution from $n = 0$, and the metric (2.4) becomes the metric for $AdS_7 \times S^4$, as in [5], which is indeed believed to be dual to the $(2,0)$ SCFT. The six dimensional Poincare symmetry is enhanced to the conformal group, and the $SO(4)$ global R-symmetry is enhanced to $SO(5)$.

The second interesting limit is large U. For $U \gg 1/l_s^2$, the sum in (2.5) can be approximated by an integral, and the result is

$$ds^2 = l_p^2 \frac{U^{2/3}}{(Nl_s^2)^{1/3}}[dx_6^2 + \frac{Nl_s^2}{U^2}(dU^2 + dy_{11}^2) + Nl_s^2 d\Omega_3^2] \,. \tag{2.6}$$

For $U \gg \sqrt{N}/l_s^2$ the theory becomes weakly coupled type IIA string theory. The quantity in the square parentheses (without the dy_{11}^2 term) is exactly the type IIA string metric, and the IIA string coupling is

$$g_s^2(U) = \frac{N}{l_s^4 U^2} \,. \tag{2.7}$$

Furthermore, for large N the curvatures are small (either in the eleven dimensional metric or in the ten dimensional metric) for any value of U, so we can use the low-energy supergravity to compute some properties of the type IIA NS5-brane theories.

We will sometimes find it useful to work, in the weakly coupled string theory regime, with a new coordinate ϕ which is

$$Ul_s^2/\sqrt{N} = e^{\phi/\sqrt{N}l_s}. \tag{2.8}$$

This brings the metric to the more familiar linear dilaton form

$$ds^2_{string} = dx_6^2 + d\phi^2 + Nl_s^2 d\Omega_3^2 \,, \qquad g_s^2(\phi) = e^{-2\phi/\sqrt{N}l_s} \,. \tag{2.9}$$

2.2. Energy scales in the theory of type IIA NS5-branes

There are several energy scales that could be important in the discussion of the decoupled theory of type IIA NS5-branes. The first scale is $1/\sqrt{N}l_s$, which is the scale that appears explicitly in the metric in the linear dilaton region. This scale also appears in previous computations of various properties of the NS5-brane theories; for instance, their Hagedorn temperature is $T = 1/\sqrt{N}l_s$.

We can find additional energy scales in the problem by examining cross-over regions as we change U. Naively we would interpret a position in the U coordinate as corresponding to an energy scale $U \sim E^2$. However, because there is another dimensionful parameter in our problem, l_s, we can also relate effects happening at some position U to physical processes at energies $E \sim \sqrt{U} f(Ul_s^2)$ for some function f. To be precise one must define the process of interest first, and the scale U may in principle appear in different ways in different processes.

As we change U, we can identify the following cross-over scales where the behavior of the theory changes :

1. One special point is where $g_s = 1 \Rightarrow U \sim \sqrt{N}/l_s^2$. At this scale we go over from weakly coupled type IIA string theory to a strongly coupled theory (*i.e.* M-theory).

2. Another scale is the place where the radius of the y_{11} circle is of the same order as the radius of the S^3, naively indicating a crossover to an $AdS_7 \times S^4$ regime of the theory. In the metric written above this happens at $U \sim 1/l_s^2$. At this scale U is of the same order as the periodicity in y_{11}, and we can no longer approximate the sum over n in (2.5) by an integral.

2.3. The type IIB NS5-brane

The behavior of type IIB NS5-branes in the limit (2.1) is similar to the IIA solution in the linear dilaton regime, but it is very different close to the 5-branes, as in the limit (2.1) the IIB solution becomes singular close to the 5-branes.

The behavior of the metric in different regimes is analyzed in [27]. The string metric far from the branes is the same as the one in (2.6), but now it is more natural to define the U coordinate as $U = r/g_s l_s^2$ which is the mass of a D-string stretched between two NS5-branes, and from it define ϕ as in (2.8). In terms of this coordinate the string metric and coupling are as in (2.9). As we decrease U, we encounter the first crossover scale at $U \sim \sqrt{N}/l_s$, where $g_s \sim 1$. At this scale the string coupling becomes large, but the curvatures (in the Einstein metric) are still small, so we can go over to an S-dual picture [27]. In the dual picture the string metric is the same as above, multiplied by $1/g_s \sim U$, and the coupling behaves as $\tilde{g}_s^2 = 1/g_s^2 \sim l_s^2 U^2/N$. In this new description, the string coupling becomes smaller and smaller as we decrease U, but the curvature (for example of the S^3) becomes larger and larger. The curvature becomes Planckian at the scale $U \sim 1/\sqrt{N}l_s$, so beyond this scale we can no longer trust supergravity. This agrees with our expectations, since the low-energy gauge coupling is given by $g_{YM}^2 = l_s^2$, so we expect perturbation theory to be valid whenever the dimensionless coupling $g_{YM}^2 NU = Nl_s^2 U$ is small (interpreting U as an energy scale in the SYM theory). Thus, we can interpret this scale as corresponding to the breakdown of the SYM perturbation theory [27].

3. Observables and correlation functions of the NS5-brane theories

The six dimensional NS5-brane theory has a scale, l_s. In the dual description this scale appears in the metric, *e.g.* as in (2.4), (2.5) for the IIA case. As discussed above, to study the long distance behavior of the theory we have to analyze it in the limit $l_s \to 0$, where it is governed by a local QFT, the $(2,0)$ SCFT for IIA fivebranes and the IR free SYM with sixteen supercharges for IIB. If the fivebrane theory had been a local QFT for all energy scales, the short distance behavior would have been governed by a UV fixed point. This fixed point would have been studied by taking $l_s \to \infty$ in (2.4), (2.5). In our case this limit leads to the linear dilaton geometry (2.9).

Because string theory in the linear dilaton geometry (2.9) cannot describe a field theoretic UV fixed point, the NS5-brane theory is not a local QFT. However, we do expect the fivebrane theory to have the property that, as in local QFT, observables are defined in the UV region (2.9). Therefore, as a check of the duality, we next discuss the spectrum of excitations of string theory in the linear dilaton vacuum and compare it to the set of observables of the NS5-brane theory. We mainly focus on short representations of supersymmetry, since the complete list of those is independently known in the fivebrane theories.

To find these short representations, recall that "little string theories" with sixteen supercharges have an $A-D-E$ classification. In the usual description the different theories may be obtained by studying decoupling limits of type II string theory on $K3$ with $A-D-E$ type singularities. In the CHS limit (2.9) they correspond to the $A-D-E$ classification of modular invariants of $SU(2)$ WZW models [21].

The global symmetry of these theories is $SO(4)$. The moduli spaces of vacua are $(S^1 \times \mathbb{R}^4)^r/\mathcal{W}$ for the type IIA theory and $(\mathbb{R}^4)^r/\mathcal{W}$ for the type IIB theory, where r is the rank of the $A-D-E$ group and $\mathcal{W}$ is its Weyl group. The global $SO(4)$ symmetry acts on the $\mathbb{R}^4$ factors. This suggests that the special chiral representations are correlated with $\mathcal{W}$-invariant products of scalars, which are the natural coordinates on the moduli space. In the type IIB theory they are easily identified as the gauge invariant polynomials in the scalars in the gauge multiplets; *i.e.* they are identified with the Casimirs of the gauge group. Their $SO(4)$ representation is a traceless symmetric tensor whose order is the order of the corresponding Casimir.

A similar result is expected in the IIA theory. This can be shown by compactifying it to five dimensions, where the low energy theory is an $A-D-E$ gauge theory. Therefore, the representations are traceless symmetric $SO(5)$ tensors whose orders are those of the Casimirs. For the A_n theories the same conclusion can be reached by using their DLCQ[7] formulation [29]. For the A, D models with large N one can also use the M-theory duals of the relevant SCFTs [30, 31, 32, 33]. Of course, only an $SO(4)$ out of the $SO(5)$ R-symmetry of the $(2,0)$ SCFT is visible at large U.

In the next subsection we discuss string theory in the linear dilaton CHS background (2.9) and show that the spectrum of short representations of supersymmetry is identical to that described above.

3.1. Worldsheet aspects of six dimensional string theory

The CHS background (2.9) is:

$$\mathbb{R}^{5,1}\times\mathbb{R}\times S^3_N\,. \tag{3.1}$$

$\mathbb{R}^{5,1}$ is the six dimensional spacetime of the $(2,0)$ theory; the remaining four dimensions parametrize the space transverse to the fivebranes. The second factor in (3.1) is the radial direction ϕ or U (2.8). It is described on the worldsheet by a free field whose stress tensor has an improvement term,

$$T_\phi=-\frac{1}{2}(\partial\phi)^2-\frac{Q}{2}\partial^2\phi\,;\qquad Q=\sqrt{\frac{2}{N}}\,. \tag{3.2}$$

The three-sphere S^3_N of radius $\sqrt{N}l_s$ describing the angular coordinates corresponds to an $SU(2)$ WZW model with level $k=N-2$ (note that this requires $N\geq 2$).

In addition to the above bosonic worldsheet fields, the theory also has free worldsheet fermions. The worldsheet fields $X^\mu(z,\bar z)$ ($\mu=0,1,2,\cdots,5$) parametrizing the six dimensional spacetime are accompanied by left and right moving superpartners $\chi^\mu(z)$, $\bar\chi^\mu(\bar z)$; the worldsheet superpartners of ϕ and the $SU(2)$ WZW are $\psi^0(z)$, $\bar\psi^0(\bar z)$ and $\psi^i(z)$, $\bar\psi^i(\bar z)$ $(i=1,2,3)$, respectively.

The $SO(4)\simeq SU(2)_R\times SU(2)_L$ isometry of the three-sphere in (3.1) acts on the worldsheet fields as follows. The bosonic $SU(2)_k$ WZW model contains (anti-) holomorphic currents $J^i_B(z)$, $\bar J^i_B(\bar z)$ which generate an $SU(2)\times SU(2)$ symmetry. The fermions $\psi^i(z)$, $\bar\psi^i(\bar z)$ transform in the adjoint of an $SU(2)\times SU(2)$ symmetry, generated by the level two currents

$$J^i_F=\frac{1}{2}\epsilon^{ijk}\psi_j\psi_k\,;\qquad \bar J^i_F=\frac{1}{2}\epsilon^{ijk}\bar\psi_j\bar\psi_k\,. \tag{3.3}$$

The total currents

$$J^i=J^i_B+J^i_F\,;\qquad \bar J^i=\bar J^i_B+\bar J^i_F \tag{3.4}$$

generate the above $SO(4)$ symmetry. The total level of the currents J^i, $\bar J^i$ is $(N-2)+2=N$.

[7] The extension to the D_n, E_n theories leads to predictions about the cohomology with compact support of the moduli spaces of D_n and E_n instantons on $\mathbb{R}^4$.

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The spacetime supercharges of the CHS near-horizon string theory are a subset of those of the full IIA string theory. Denoting by $\alpha \in \mathbf{4}, \bar{\alpha} \in \bar{\mathbf{4}}$ and $a \in \mathbf{2}, \bar{a} \in \bar{\mathbf{2}}$ spinor indices of $SO(5,1)$ and $SO(4)$, respectively, the thirty two supercharges of IIA string theory transform as $Q_{\alpha a}$, $Q_{\bar{\alpha}\bar{a}}$, $\bar{Q}_{\alpha\bar{a}}$, $\bar{Q}_{\bar{\alpha}a}$ (Q, $\bar{Q}$ arise from left, right movers on the worldsheet, respectively). By using the form of the gauged worldsheet $\mathcal{N}=1$ superconformal generators:

$$\begin{aligned} T &= -\frac{1}{2}(\partial X^\mu)^2 - \frac{1}{2}\psi^\mu\partial\psi^\mu + T_\phi - \frac{1}{N}J^iJ^i - \frac{1}{2}\partial\psi^a\psi^a \\ G &= \psi^\mu\partial X^\mu + \psi^0\partial\phi + \sqrt{\frac{2}{N}}(\psi^i J^i + \psi^1\psi^2\psi^3 - \partial\psi^0) \end{aligned} \tag{3.5}$$

and the similar formulae for the other worldsheet chirality, one can show that the physical supercharges in the background (3.1) are $Q_{\alpha a}$, $\bar{Q}_{\alpha\bar{a}}$, which generate $(2,0)$ six dimensional SUSY.

Note that we could have studied the CHS limit of NS5-branes in type IIB string theory as well. In that case the surviving SUSY would have been $(1,1)$ and we would have obtained the other six dimensional string theory discussed in [15]. The low energy limits of the $(2,0)$ and $(1,1)$ string theories are very different. While the former flows in the infrared to the non-trivial $(2,0)$ field theory, the latter reduces at low energies to the (infrared free) six dimensional SYM. From the point of view of the string theory in the tube limit (3.1), this difference has to do with the different ways the IIA and IIB theories treat the strong coupling region at small ϕ.

The following general features are clear from the above description.

1. The NS5-brane theory has a stress tensor. In the (dual) string theory of the CHS tube this is the statement that the theory has six dimensional gravitons (generalizing the identification of the graviton with the energy-momentum tensor in the AdS/CFT correspondence [7, 8]).

2. Upon compactification on tori, the NS5-brane theory has T-duality [15]. In our description this arises from the T-duality of the dual IIA or IIB string theory. Thus, it cannot be a local QFT. In particular, upon compactification the identification of the graviton is not unique (since it varies when we T-dualize), corresponding to the non-uniqueness of the energy momentum tensor in the six dimensional theory [15].

We next turn to the spectrum of physical operators of the string theory in the CHS tube. We will focus on short representations of the relevant SUSY algebra. As a first example consider the A series of six dimensional string theories, which is conjectured to be dual to the theory with the A modular invariant of $SU(2)$ in the tube limit. The only primaries of $SU(2)_R \times SU(2)_L$ that appear in the A modular invariant of $SU(2)_k$ are $V_{j,j}$ with spin j for both $SU(2)$'s, with $2j = 0, 1, 2, \cdots, k$ (recall that the level k is related to the number of fivebranes N via $k = N - 2$). Each such primary gives rise to a short representation. As is clear from our previous comments on the global symmetries, the state with spin j transforms as a traceless symmetric tensor with $2j$ indices under $SO(4)$.

To obtain the bounds on j we need to describe the physical states slightly more precisely. The lowest component of each representation is a scalar under six dimensional Lorentz. The corresponding vertex operator takes the form (in the -1 picture)

$$\psi\bar{\psi}V_{jj}e^{\beta_j\phi}\,. \tag{3.6}$$

The fermions ψ, $\bar{\psi}$ transform as the $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ of $SU(2)_L\times SU(2)_R$; $\beta_j=j\sqrt{\frac{2}{N}}$. The indices under each $SU(2)$ are contracted between the fermions and V_{jj} to form representations with spin $j-1$, j and $j+1$. One can show that the representation with spin j is unphysical, while that with spin $j+1$ gives rise to the lowest component of a short representation. The states with spin $j-1$ are descendants in this representation. Thus, one finds short multiplets in the $SU(2)_R\times SU(2)_L$ representations of spin $(j+1,j+1)$ with $2j=0,1,2,\cdots,N-2$ as above. In $SO(4)$ language these are traceless symmetric tensors with $2j+2=2,3,\cdots,N$ indices. This agrees with our general expectation in terms of the Casimirs of A_{N-1}. Note that in our description we can explicitly see the truncation of the chiral operators at $2j+2=N$, which is generally obscure in the AdS/CFT correspondence. This provides further evidence for the validity of the bulk/boundary correspondence for finite values of N.

Returning to the $SO(4)$ vs. $SO(5)$ issue, we expect (just like in type IIA string theory in flat spacetime) the symmetric tensors of $SO(4)$ to be extended to symmetric tensors of $SO(5)$ with non-perturbative states in the CHS string theory, *i.e.* D-branes. The missing states are exactly the D0-brane states, which indeed exist only for type IIA. Adding these states we find, for large N, the full spectrum of the low-energy SCFT. We have not shown that for finite N the spectrum of states involving D0-branes truncates at the appropriate value of j.

The D-series models exhibit some new features. Recall that D-series modular invariants in $SU(2)$ WZW CFT exist only for even k and they have the following spectrum of primary operators:

1. Operators $V_{j,j}$ with equal left and right spins which are both *integer* (and as usual bounded from above by $k/2$).

2. A single additional operator $V_{j,j}$ with $j=k/4$.

3. A series of operators V_{j_1,j_2} with $j_1=\frac{k}{4}+n$ and $j_2=\frac{k}{4}-n$ with integer n.

The corresponding low energy theory is an $SO(2N)$ gauge theory with sixteen supercharges for type IIB, and the D_N $(2,0)$ SCFT (which is dual to M-theory on $AdS_7\times RP^4$ [30]) for type IIA (the relation between k and N is in this case $k=2(N-2)$). The first kind of operators in the tube string theory above are interpreted as before in the two low energy theories. The operator (2) above is also easy to interpret: it corresponds to the Pfaffian operator in the $SO(2N)$ gauge theory that appears for type IIB, and to a similar operator of dimension $\Delta = 2N$ in the $(2,0)$ SCFT for type IIA. In M-theory on $AdS_7 \times RP^4$ this operator corresponds to an $M2$-brane wrapped around an RP^2 in RP^4 (as in [34]).

The operators (3) are interesting. Since their worldsheet left and right scaling dimensions differ by an integer they are similar to Dabholkar-Harvey [35] states in toroidal compactifications of string theory and, just like the above, they give rise to a large number of "medium" multiplets. It would be interesting to understand whether supersymmetry is sufficient to guarantee their appearance in the low energy theory, and, if the answer is yes, to identify them in the low energy IIA and IIB theories.

One can also study the E_6, E_7 and E_8 six dimensional string theories. These theories have the following structure in the tube limit. The bosonic WZW model contains operators with $SU(2)_R \times SU(2)_L$ spin (j,j) with the following values of $2j$:

$$\begin{aligned} E_6 :&\quad 2j = 0,3,4,6,7,10 \\ E_7 :&\quad 2j = 0,4,6,8,10,12,16 \\ E_8 :&\quad 2j = 0,6,10,12,16,18,22,28. \end{aligned} \tag{3.7}$$

Each of these gives rise, as in the A, D series above, to short multiplets in the six dimensional string theory. At low energies the spectrum of chiral primaries should be compared to the six dimensional IR free gauge theory with sixteen supercharges and E_n gauge group for type IIB, and to the E_n (2,0) SCFT for IIA. The latter cannot be described by eleven dimensional supergravity since the curvature of the relevant eleven manifold is large in Planck units, but our general considerations above suggest that again the special representations are traceless symmetric $SO(5)$ tensors with the number of indices related to the order of the E_n Casimir. It is easy to see that the results predicted by (3.7) are indeed correlated with these Casimirs in the right way.

The analysis of [21] gave an explanation of the origin of the $A-D-E$ classification of affine $SU(2)$ modular invariants. The results here explain another aspect of this classification which is not obvious from purely conformal field theory considerations. It clarifies why the spinless primaries are correlated with the Casimirs of $A-D-E$ and, therefore, why their number is given by the rank of the $A-D-E$ group.

The E_n theories, just like the D_n ones, have states $V_{j\bar{j}}$ with $j \neq \bar{j}$. Thus the issue of the existence of medium representations in the infrared theory raised in that context must be understood here as well.

3.2. Branes in the near-horizon geometry

In addition to the observables described in the previous subsection, branes can also propagate in the backgrounds described above. It is interesting to analyze the brane spectrum in the geometry we find, and to interpret it in terms of the dual NS5-brane theory.

Let us start with the small U limit of the IIA case, where the theory goes over to the $AdS_7 \times S^4$ compactification of M-theory, which is conjectured to be dual to the $(2,0)$ 6D SCFT. The interesting branes in this limit seem to be the membrane and the M-theory 5-brane wrapped around S^4. The membrane tension does not depend on the U coordinate; the U coordinate is defined so that the mass of a membrane stretched in the U direction is linear[8] in U. This enables us to look at configurations of membranes ending on some surface at the boundary of AdS (large U); the energy of these configurations is linearly divergent in U, and they may be identified with "Wilson surface" observables of the $(2,0)$ SCFT.

On the other hand, a 5-brane wrapped on S^4 cannot exist on its own, like the wrapped 5-branes in the $AdS_5 \times S^5$ background [34]. This is because the background 4-form field acts as a source of 2-form charge in the 5-brane, which must be balanced by N M theory membranes ending on such a 5-brane in the AdS_7 (note that the wrapped 5-brane is a string in the AdS_7). These membranes must have another boundary, so they must stretch to the boundary of AdS (or to another 5-brane with opposite orientation); thus this configuration behaves like a baryon which contributes to the product of N "Wilson surface" observables.

Both types of branes described above exist also in the full background described in the previous section. In the linear dilaton region, the membrane becomes a D2-brane of type IIA string theory, whose tension (in 11D Planck units) is still independent of U. These membranes/D2-branes may be used to define "Wilson surface" observables of the type IIA NS5-brane theories, by analyzing configurations with D2-branes stretching to infinity in the U direction. At large distances (measured in string units) these observables will go over to the "Wilson surface" observables of the low-energy $(2,0)$ SCFT [36].

Similarly, the 5-brane wrapped on S^4 becomes a D4-brane wrapped on S^3. The background NS 3-form field induces a magnetic charge in the 4-brane field theory, which must be balanced by N D2-branes, as before. So, this state still behaves as a generalized baryon vertex.

We will not discuss here the most general possible branes, but it is interesting to analyze D-branes which stretch only in the $\mathbb{R}^{5,1}$ directions (in the linear dilaton region). The mass of such D-branes behaves (in string units) like $1/g_s$, so they are not stable objects but instead tend to fall into the strong coupling region. This agrees with the expectation [15] that D-branes in $\mathbb{R}^{5,1}$ will form bound states with the NS5-branes; we can interpret the branes falling in to small values of U as a bound state of the D-branes with the NS5-branes, which tends to spread out in the $\mathbb{R}^{5,1}$ directions (using the interpretation of U as an inverse distance scale).

The analysis is similar for the type IIB theory. In this theory the D1-brane has (by construction) finite tension for large U, and can be used to define Wilson line operators (which go over to Wilson lines in the 6D SYM theory at large distances). A D3-brane

[8]Note that the coordinate U we use here is the square of the coordinate U in [5].

wrapped around the S^3 serves as a baryon vertex, since N D-strings have to end on such a D3-brane. D-branes in $\mathbb{R}^{5,1}$ can be interpreted as in the previous paragraph. For very small values of U it should be possible to interpret the fundamental string (stretching in the $\mathbb{R}^{5,1}$ directions) as an instanton of the SYM theory, and the U coordinate as the scale size of this instanton (this is similar to the interpretation of D-instantons in the $AdS_5 \times S^5$ background [37, 38, 39]), but it is not clear if such an interpretation makes sense also for large U.

3.3. Holographic computation of a scalar correlator in the IIA NS5-brane theory

In this subsection we will discuss some aspects of the Euclidean 2-point function of a scalar field in the theory of the IIA NS5-branes. We will do the computation in low-energy supergravity, implying the following limitations:
1. The momentum has to be well below the string scale $p \ll 1/l_s$. Otherwise there could be large corrections to the supergravity analysis.
2. We require that N be large so that the various curvatures in the solution are small.

Even under these restrictions there are two distinct regimes of momenta, compared to the scale $1/\sqrt{N}l_s$. At low momenta compared to this scale we will reproduce the results of the low-energy $(2,0)$ SCFT, whereas for $p > 1/\sqrt{N}l_s$ (but still much smaller then $1/l_s$) we will start seeing the effects of the "little string theory".

Note that we do not really have a scalar field in the problem (which is a scalar in 11D), but this simpler computation should have the same qualitative features as the correlation functions of other fields in the theory (such as the graviton, which is related to the six dimensional energy momentum tensor).

We will follow the procedure of [7] for computing 2-point functions of scalar fields. We should start by finding the solution to the equation of motion for a scalar field in the background (2.4). Assuming no dependence on the S^3 coordinates, and an $e^{ip\cdot x}$ dependence on the $\mathbb{R}^{5,1}$ coordinates, the equation of motion is

$$[\partial_U(U^3\partial_U) + U^3\partial_{11}^2 - \widetilde{H}U^3p^2]\Phi(U, y_{11}) = 0\,, \tag{3.8}$$

where $\widetilde{H}$ was defined in (2.5).

We have not been able to solve (3.8) exactly, but dimensional analysis of this equation reveals much of its physical content. The factor of N appears only in the last term, hence the dependence of the solution on p and on N will be via the combination p^2N. Furthermore, if we rescale U and y_{11} by l_s^2, making them dimensionless, then the differential operator in (3.8) depends only on $Nl_s^2p^2$. This is true in all the regions of the SUGRA background. If we are interested in momenta below $1/\sqrt{N}l_s$, then we may hold p fixed and take $l_s \to 0$, in which case the differential equation becomes that of a scalar field on $AdS_7 \times S^4$ and we will reproduce the results of the $(2,0)$ SCFT. When l_s is not strictly zero, the correlator will deviate from that of the SCFT, and this deviation will be governed in momentum space by powers of $Nl_s^2p^2$. Thus, eleven dimensional supergravity can be used to analyze the behavior of the NS5-brane theory away from the IR fixed point, for $1/\sqrt{N}l_s < p \ll 1/l_s$.

In the low momentum regime $p \ll 1/\sqrt{N}l_s$ we can make the computation more explicit. This will also serve to show that the dependence on the UV cutoff (which one takes to infinity only at the end of the computation of a correlator) does not invalidate the conclusion of the previous paragraph. In the large U region the radius of the y_{11} direction vanishes and therefore $\partial_{11}\Phi = 0$. Thus, we get the equation

$$[\partial_U(U^3\partial_U) - Nl_s^2p^2U]\Phi(U) = 0\,, \tag{3.9}$$

which has two independent solutions, $\Phi_\pm \sim U^{\beta_\pm}$, where $\beta_\pm = -1 \pm \sqrt{1+Nl_s^2p^2}$.

In the small U, y_{11} region (the $(2,0)$ SCFT region) the metric looks like $AdS_7 \times S^4$, so we have the standard solution for a scalar field in AdS_7. The solution on this space which is regular at $U=0$ is

$$\Phi_0(U, y_{11}) = (\sqrt{Np^2}z)^3 K_3(\sqrt{Np^2}z), \tag{3.10}$$

where $z = (U^2 + y_{11}^2)^{-1/4}$ and K_3 is a Bessel function. In particular, the solution in this regime has an asymptotic expansion in $\sqrt{Np^2}z$, of the form

$$\Phi_0(z) \sim 1 + a_1 Np^2z^2 + a_2(Np^2z^2)^2 + a_3(Np^2z^2)^3(log(Np^2z^2) + a_4) + \cdots \tag{3.11}$$

for some constants a_i. This asymptotic expansion can be used when both $\sqrt{Np^2}z \ll 1$ and $U \ll 1/l_s^2$, which implies that $p^2 \ll 1/Nl_s^2$, so we are at very low momenta where we expect the result to be similar to the result for the $(2,0)$ SCFT.

As in [7], we will set a cutoff U_0 for the theory, and compute the 2-point function with this cutoff, taking $U_0 \to \infty$ at the end. For $U_0 \gg 1/l_s^2$, the solution near U_0 will be some linear combination of the solutions $\Phi_\pm$ described above, which is determined by the fact that it should go over to Φ_0 for small U. Thus, the solution for large U will be of the form

$$\Phi(U) \sim \frac{a_+(p)U^{\beta_+} + a_-(p)U^{\beta_-}}{a_+(p)U_0^{\beta_+} + a_-(p)U_0^{\beta_-}}, \tag{3.12}$$

where $a_\pm(p)$ are some functions which are determined by the form of the exact solution, and we normalized the solution so that $\Phi(U_0) = 1$ (as in [7]).

As in [7], we would now like to evaluate the action for the scalar field. It reduces to a boundary term at $U = U_0$, which is of the form $F(p) \propto U^3\Phi\partial_U\Phi$ (evaluated at $U = U_0$). Substituting in the solution (3.12), we find that for large U_0 this behaves like

$$F(p) \propto U_0^2(\beta_+ + (\beta_- - \beta_+)\frac{a_-(p)}{a_+(p)}U_0^{\beta_- - \beta_+} + \cdots)\,. \tag{3.13}$$

The two point function in position space will be given by the Fourier transform of the leading non-analytic term in $F(p)$ (in the $U_0 \to \infty$ limit). The expansion (3.11) suggests that we can expand the whole solution in an asymptotic expansion in p^2, for which the leading non-analytic term would be at the order $p^6 \log(p^2)$, with additional non-analytic terms of the form $p^6(p^2 N l_s^2)^n \log(p^2)^k$ (times some function of U). This was shown to be true for the solution in the full 3-brane metric which interpolates between $AdS_5 \times S^5$ and flat Minkowski space in [40], and we expect a similar behavior here. Thus, the function $a_-(p)/a_+(p)$ should have a similar asymptotic expansion, so that the leading non-analytic term in (3.13) is of the order of $p^6 \log(p^2)$ (note that this leading term is independent of U_0), leading to a 2-point function which behaves like $1/|x-y|^{12}$. This is exactly the behavior we expect for a dimension 6 operator (corresponding to a massless scalar field in supergravity) in the low-energy SCFT. The procedure above enables us to compute also the first corrections to this low-energy expression, and we immediately see that they will depend on $Nl_s^2/|x-y|^2$, as expected.

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